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Counting and measuring and approximation

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1. Introduction: the problem

In previous talks, I have argued extensively that mass nouns allow quantity evaluations and comparisons either in terms of cardinality or along a dimensional scale, while count nouns (almost always – see last section) require comparison in terms of cardinality (1). These results are replicated in Brazilian Portuguese as in (2) (Pires de Oliveira and Rothstein 2011), Hungarian (3), (Schvarcz and Rothstein to appear), and Mandarin (4) (Rothstein in press), as well for the determiner *most* in English (5) (Landman 2011):

(1) a. John has more furniture than Bill so he will need the bigger truck.
b. John has more pieces of furniture than Bill, so he will need the bigger truck.

(2) a. **Quantos** **livro-s** ele compr-ou? (PdeO&R 2011: 52b)
how.many book-PL he buy-PST.PRF.3SG
'How many books did he buy?'
b. **Quanto** **livro** você compr-ou?
How.much book you buy-PST.PRF.3SG
'What quantity of books did you buy?'

(3) a. Hány könyv van a táská-d-ban? (Schvarcz & Rothstein to appear)
How.many book is there the bag-POSS.2SG-in
'How many books are there in your bag?'
(.i) Csak három. (ii). # Három kiló.
Only three. Three Kilo
'Only three.' 'Three kilos.'

b. Mennyi könyv fér a táská-d-ban?
How.much book. fit.PRES.3SG the bag-POSS.2SG-in?
'What quantity of book fits into your bag?'
(i) Három kiló-t (ii) Hárm-at
Three kilo-OM three-OM
'Three kilos.' 'Three'

(4) nǐ dài tài duō (běn) shū le, xínglǐ huì chāozhòng de
you take too much/many Cl_{volume} book PRF baggage will overweight PRT
'You have taken too many books, your baggage will be overweight.' (Rothstein in press)

(5) a. Most farm animals are chickens. (Examples based on Landman 2011)
b. Most farm animals are cows.
c. In terms of **number**, most livestock is poultry.
d. In terms of **volume**, most livestock is cattle.
e. #In terms of volume, most farm animals are cattle.

Previous talks focused on the data. Here I want to focus on the following question:
How can we compare two quantities in terms of their cardinality, if they are not countable? or, more precisely**:**
How can *more* or *most* apply to non-count predicates, and give a meaning which compares in terms of cardinality?

2. Background: a theory of mass/count (Rothstein 2010)

Count nouns:

- Counting is **context dependent**: we count, in a particular context k, the entities which in that context are considered atomic entities. The context dependence of count noun denotations (and of counting) is based on the observation in Rothstein (2010) that with nouns like *fence*, *wall*, *place*, *hedge*, as well as *thing*, and *object*, what counts as an atomic entity, i.e. a single *wall* or *fence* is context dependent. One stretch of fencing can count as one fence or several fences depending on your choice of what counts as 'one'.

- Counting is **counting of discrete entities**, atoms, portions. It uses a non-continuous set of numbers.

- The counting operation**, the cardinality function,** which maps a sum on the number of its atomic parts, is relative to a particular context k (since first you need to know what counts as 'one' before you can start counting.

Count nouns are grammatically countable and can be modified by cardinals because they encode the contextual parameter k. They denote sets of atoms (or pluralities of atoms) indexed for the context in which they count as atomic. Count nouns are of type $<e\times k, t>$, and denote sets of entities of type $e\times k$, where k is the relevant index.

The standard cardinality operation is given in (6). We modify it to (7) expressing the fact that counting is always relative to a context k. The k-atoms of x are the minimal parts of a k-indexed entity.

(6) $|\ x\ | = n \leftrightarrow |\{y: y \sqsubseteq_{ATOMIC} x\}| = n$
"The cardinality of a sum *x* is *n* if the cardinality of the set of the atomic parts of *x* is *n*"

(7) a. $|\ x\ |_k = n \leftrightarrow |\{y: y \sqsubseteq_{k\text{-}ATOM} x\}\ | = n$
b. $y \sqsubseteq_{k\text{-}ATOM} x$ iff y is a minimal part of x in context k
"The cardinality of a sum *x* in context *k* is *n,* if *x* is *n* if the cardinality of the set of the atomic parts of *x* in context *k* is *n*"

(8) $y \sqsubseteq_{k\text{-}ATOM} x$ iff $x \in D\times K \wedge y \sqsubseteq x \wedge \forall z\, [z \sqsubseteq x \wedge z \sqsubseteq y \rightarrow z=y]$

Mass nouns denote sets at the simple set type $<e,t>$, i.e. sets of entities. The entities in the denotations of mass nouns may or may not be naturally atomic. Since lexical counting i.e. modification by cardinals, is sensitive to the choice of k parameter, mass nouns cannot be counted. But they can be measured.

Derivations: Mass noun and count nouns are derived from root nouns.
Root nouns are predicates at type $<e,t>$.

MASS(N_{root}) is the identity operation on root nouns. N_{mass} is a predicate of type <e,t>. It denotes a property of individual entities (or sums).
COUNT(N_{root}) is an operation which maps N_{root} onto N_k, of type $<e \times k, t>$, denoting the set of (indexed) entities which count as atoms in context k, i.e. a set of ordered pairs <x, k>, where x is an entity in N and k is the context. They denote properties of indexed individuals. They are semantically atomic.

Examples: $[\![stone_{mass}]\!] = MASS(STONE_{root}) = STONE_{root}$
$[\![stone_{count}]\!] = COUNT_k(STONE_{root}) = \{<d, k>: d \in STONE_{root} \cap k\}$

So *stone*$_{mass}$ denotes a set of quantities of stone, while *stone*$_{count}$ denotes a set $\{<d, k>: d \in STONE_{root} \cap k\}$ of type $< e \times k, t>$ i.e. the set of indexed entities which count as one in context k.

3. Our problem: how do we give quantity evaluations of sums in the denotation of mass nouns if we can't count them?

Here is the problem again: Look at (9). (9) is true iff the cardinality of the sum of books that Mary has is greater than the cardinality of the sum of books that John has.

(9) Mary owns more books than John

We assume at this point a cardinal semantics for *more*. *More* denotes an operator $MORE_{|\ |k}$ of type <e <e,t>>, which maps pairs of entites <x,y> on to truth values, with the value true if the cardinality of x is greater than the cardinality of y in context k, otherwise false, i.e. (10):

(10) x is $MORE_{|\ |k}$ than y iff $|\ x\ |_k > |\ y\ |_k$

(9) is true if (11) holds.

(11) $|\sqcup(\text{BOOKS} \cap \text{OWNED BY MARY})\ |_k > |\sqcup(\text{BOOKS} \cap \text{OWNED BY JOHN})\ |_k$

By extension, you would expect (12a) to be true on the cardinality reading iff the cardinality of the sum of furniture that Mary has is greater than the cardinality of the sum of furniture that John has, i.e. (12b):

(12) a. Mary owns more furniture than John
b $|\sqcup(\text{FURNITURE} \cap \text{OWNED BY MARY})\ |_k > |\sqcup(\text{FURNITURE} \cap \text{OWNED BY JOHN})\ |_k$

On the assumption that mass nouns are not generated by sets of k-atoms the cardinality function should not be able to apply to sums in the denotation of mass nouns since it makes reference to atomic parts. This makes (12b) impossible as the interpretation of (12a). This problem is not specific to the particular theory of count nouns I am using. Any theory which assumes that mass nouns are not generated by sets of atoms will face this problem since the cardinality function counts atoms.

Possible solution 1: (9a) is evidence that the cardinality function does apply to mass nouns, and therefore object mass nouns have the same semantics as count nouns (i.e. denote sets of atoms) (though not the same syntax).. This is Bale and Barner's (2009) solution.

Possible solution 2: The cardinality function does not apply to mass nouns, and therefore (12b) cannot be the right interpretation for (12a). Something else must be going on.

Reasons for going with possibility (2):

1. As I showed in my previous work (Rothstein in press) and as shown experimentally in Grimm and Levin (2012) (replicated in Gafni and Rothstein 2014), mass nouns allow comparison by cardinality but also allow comparison along other continuous dimensions, whereas count nouns (almost always) force cardinal comparisons. If cardinal comparisons indicate count denotations, then object mass nouns would have to have two different denotations; a count denotation which licensed the cardinal comparison and a mass denotation which licensed the non-cardinal comparison. This seems implausible. So the cardinal comparisons must work differently in the mass domain from in the count domain.

2. If the cardinality function does apply in the mass domain, then there is no good reason why cardinals shouldn't modify mass nouns. But they don't. Bale and Barner's solution: cardinals select for syntactically count nouns. But this is equivalent to suggesting that 'count noun' is a syntactic category and does not reflect a particular semantics or set of semantic properties. For Bale and Barner object mass nouns and count nouns have the same kind of semantic representation (although different derivational histories). Cardinal numerals are sensitive to the syntactic features and not the semantic representation.
If we want a semantic definition of countability, this solution doesn't work.

3. Landman 2015: Object mass nouns (neat mass nouns) are not generated by a disjoint base since the cup, the saucer and the tea-set which includes them can all be considered single items of crockery. So applying the cardinal function to sums in the denotation of object mass nouns should not be possible.

4. Rothstein 2010, counting is relative to a contextual parameter k.
k is the set of objects that count as one (i.e. atomic) in context k.
The cardinality function is: $|\ \ |_k$ is defined in terms of $|\ \ |$
Since mass nouns are not k-indexed, the cardinality function can not apply to mass nouns.

So we need to ask the following questions about comparatives of object mass nouns like (12):
a. How do we represent the truth conditions of these comparatives if we can't use the cardinality function?
b. What does it mean to compare quantities in terms of cardinality if we can't count the quantities? What operation will do this for us?
C How do we explain why object mass nouns allow both cardinality evaluations and evaluations along a dimension of measure when count nouns do not?

4. Counting and measuring (based on Rothstein, in press):

Counting and measuring are two different operations:
Counting: is putting entities in one-to-one correspondence with the natural numbers. It presupposes a discrete set of non-overlapping atoms.
Measuring: is assigning to a quantity an overall value on a dimensional scale.

Counting: is putting atomic entities in one-to-one correspondence with the natural numbers. Counting gives a value to a plurality α by assigning numbers from the sequence of natural numbers in order to the atomic parts of α. α has value n if the start and final numbers are n places apart. The semantic operation which assigns a cardinal value to a plurality is the cardinality function $|\ |_k$ repeated here from (7):

$\forall x: |\ x\ |_k = n$ iff $|\{y: y \sqsubseteq_{k\text{-ATOM}} x\}|_k = n$
The cardinality of *x* in context *k* is the cardinality of the set of entities which are minimal *k*-indexed parts of *x*. i.e.
The cardinality of *x* in context *k* is *n* if the set of atomic parts of *x* (relative to *k*) has *n* members.
Counting is a function into a discrete set of values.
Cardinal modifiers are predicates s are functions at type <<e×k, t> <e×k,t>>.
Thus mass nouns at type <e,t> cannot be modified by cardinals.

Measuring is assigning a quantity a value on a scale:

A ***scale*** $S_{M,U}$ is a partial order $S_{M,U}$ = <**N,** $\geq_{M,U}$.$\text{MEASURE}_{M,U}$ > where:
M is a dimension (e.g. volume, weight).
U is the unit of measurement in the relevant dimension, in terms of which the scale is calibrated (e.g. *litre, kilo,*)
N is the real numbers, or the positive real numbers, or a subset of the real numbers, depending on the nature of the measure and the fine-grained of the measurements.
$\text{MEASURE}_{M,U}$ is a function from objects to values in N.

Because the range of values is the set of the real numbers, measuring is inherently **continuous** (see Landman, this conference). We make it non-continuous by choosing a subset of the real numbers as our contextually relevant range of values. Our choice of values determines the **granularity** of the scale. (Solt 2015)
A **measure head** such as *litre* has the denotation in (13a):
A ***measure predicate*** expresses the property of having a particular measure value, (13b):

(13) a. ⟦litre⟧ = $\lambda n \lambda x.\ \text{MEAS}_{\text{VOLUME, LITRE}}(x) = n$
b. ⟦3 litres⟧ = $\lambda x.\ \text{MEAS}_{\text{VOLUME, LITRE}}(x) = 3$
c. ⟦3 litres wine⟧ $\lambda x.\ \text{WINE}(x) \wedge \text{MEAS}_{\text{VOLUME, LITRE}}(x) = 3$

Measuring focusses on the properties of the quantity as a whole. If a quantity x measures three litres, we know nothing about its internal structure. We only know that any way we break a into two non-overlapping parts b and c, the following will hold:

If $a = b \sqcup c$ and $\text{MEASURE}_{\text{VOLUME,LITRE,}}(a) = 3$, then
$\text{MEASURE}_{\text{VOLUME,LITRE,}}(b) = 3 - \text{MEASURE}_{\text{VOLUME,LITRE,}}(c)$

5. Quantity comparisons

Quantity comparisons are either in terms of cardinality or along a continuous dimension. Some terms cue for one or the other, e.g. *how much/how many* in (14a/b) or *less* vs. *few* in (15a/b). Other terms do not cue, e.g Hebrew *kama* 'how much/many' (16a/b) or *more* (16c). In such cases, the parameter for comparison is constrained by whether N is mass or count.

(14) a. How much#many wine did you drink?
b. How many/#much bottles of wine did you drink?

(15) a. John drank fewer/#less bottles of wine than Mary
b. John drank less/#fewer wine than Mary.

(16) a. kama bakbukey yayin šatit?
KAMA bottles of wine you-drank
'How many bottles of wine did you drink?'

b. kama yayin šatit?
KAMA wine you-drank?
'How much wine did you drink?'

c. Who listened to more music/more pieces of music?

When the noun is count, the atomic structure encoded in the denotation makes the atoms salient and comparison must be in terms of cardinality. *Who has more books/pieces of furniture?* requires a comparison of the value of the cardinality function applied to each plural object (or set). If John has five folding chairs, and Mary has a bed, a piano, a table and a chair, then John has more pieces of furniture than Mary. The difference can be expressed using cardinal modifiers, as above, repeated here

(17) *Mary has more books than John* is true iff the cardinality of the sum of books that Mary has is greater than the cardinality of the sum of books that John has.

(18) x is $\text{MORE}_{|\,|_k}$ than y if $|\, x \,|_k > |\, y \,|_k$

You don't always count in order to answer *Who has more books? How many people are there in the room? Do you have more books in your living room in Tel Aviv or your living room in Tübingen?* You can estimate, or calculate. But the answer always involves a comparison of cardinalities, and the correct answer is always in terms of which counting value is higher in the sequence of natural numbers as in (18).

When the noun is mass, we compare two values on a dimensional scale to see which is higher, as in (19). (20) gives the general case.

(19) Mary drank more wine than John is true iff
$\text{MEASURE}_{\text{VOLUME},\text{U}}$(wine that M. drank) > $\text{MEASURE}_{\text{VOLUME},\text{U}}$(wine that J. drank)

(20) x is $\text{more}_{\text{M,U}}$ than y if $\text{MEASURE}_{\text{M,U}}(x) > \text{MEASURE}_{\text{M,U}}(y)$

So crucially *more* can evaluate either on a dimensional scale or in terms of cardinality.
We can give a unified meaning as in (21), where *f* is either the cardinality operation or a measure operation:

(21) $[\![\text{more}_{<e,<e,t>>}]\!] = \text{MORE}_f(x,y)$
For some function f: $\text{MORE}_f(x,y)$ iff $f(x) > f(y)$

The difference then between MORE applied to count nouns and MORE applied to mass nouns is not in the meaning of *more* which compares the magnitude of the values which are the output of *f*, but in the type of function *f* which gives you those values and thus in the type of the values.

When *more* applies to count nouns, the f function is the counting operation $|\ \ |_k$, which gives cardinality values to compare, i.e. entities of type n.
When more applies to mass nouns, the f function is a measure operation, $MEAS_{DIM,U}$ which gives measure values to compare, i.e. entities of type $n \times U$.

We cannot reduce comparison of measure values to a cardinality comparison because in measuring, values are compared relative to a specified unit, i.e. we need U to guarantee that the units in each case are commensurate. It is false that *12 inches of yarn < 20 cm of yarn*, despite the fact that 12 < 20.

7. Our original question: how do we comparing object mass nouns in terms of cardinalities?

When a noun is mass, comparison is along any contextually relevant dimension. Crucially this can include cardinality.

(22) Mary has more furniture than John.

How do we represent the truth conditions of (22) so that: (i) we allow for cardinal comparisons; (ii) we allow cardinality to be only one among a set of possible parameters of comparison. We cannot by hypothesis directly use cardinal MORE (18) because we cannot directly assign cardinal values to pluralities in the denotations of mass nouns, i.e. we cannot directly use $|\ x\ |_k$ since it presupposes a set of grammatically indexed k-atoms .

(7) $|\ x\ |_k = n \leftrightarrow |\{y : \langle y,k\rangle \sqsubseteq_{MINIMAL} x\}| = n$
The cardinality of *x* in context *k* is the cardinality of the set of entities which are minimal *k*-indexed parts of *x*.

So we cannot assign a cardinality to the sum of furniture that Mary has and the sum of furniture that John has and directly compare the values. Instead we need truth conditions for (22) that fit the template in (20) and allow for cardinal and non-cardinal evaluations.

Note I am not asking how you **decide** whether (22) is true i.e who has more furniture? You may decide by counting pieces of furniture. I am asking what the sentence you use to express your conclusion means, i.e. how you represent the truth conditions of (22), i.e. how does *more* work.

Proposal: cardinalities can be compared using values on a **cardinality scale**.
I have been assuming that numbers are objects of type n and that the natural numbers are a set totally ordered by the successor function. Counting uses this order.

However, we can treat cardinal values as if they are values on a measure scale $S_{M.U.}$

Sum x is has a cardinality value which is more than sum y if the value assigned to x is higher on the cardinality scale than the value assigned to y.
Crucially, how we assign the value is (semantically) irrelevant to the measure function, just as it is irrelevant how we measure out *three kilos of flour*.

We can compare values of sums by covert counting, by approximation, etc. But using a cardinal 'measure' operation does not require counting, since it only gives a value to the overall sum and does not individuate atomic parts.

We construct a **cardinality scale** analogous to $S_{M,U}$ from a set of numbers as follows:

A cardinality scale is an order $S_{\mathbf{CARD},|\,|^{\mathbf{k}}} = \langle \mathbf{N}, \geq_{N.} |\,|^{\mathbf{k}} \rangle$

where CARD stands for cardinality,
k is the context that determines the set of atoms,
N is the set of natural numbers,
and $|\,|^{\mathbf{k}}$ is the function that maps x onto the values in N depending on the cardinality of the set of minimal parts of x relative to k i.e. $|\{y: y \sqsubseteq x\} \cap k|$

N is the set of natural numbers since the values are not continuous.
The scale is not assigned a dimension, and any sequence can be used to model the natural numbers. See Wiese 2003 for discussion.) We can use the set k, judiciously chosen, to model the units in terms of which the scale is calibrated, since we use it anyway in $|\,|^{\mathbf{k}}$.
$|\,|^{\mathbf{k}}$ gives a cardinal value to x not by counting the atomic parts of x, but by counting a related set $\{y: y \sqsubseteq x\} \cap k$ (capturing the fact that the discrete minimal objects in the denotation of object mass nouns may be are contextually determined, as pointed out in Landman 2016).
A value on a cardinality scale thus gives you a number of units, but does not specify a dimension, and thus the information is purely quantitative.

Crucially, the cardinality function used here is not the same as in (7).
The basic cardinality 'counting' operation $|\;|_k$ directly counts the k-indexed parts of x via individuating the atoms.
The function we use here, $|\;|^{\mathbf{k}}$ used here counts indirectly.
It gives a value to x by taking the set of parts of x, intersecting this set with k and counting the resulting set i.e. $|\{y: y \sqsubseteq x\} \cap k|$
This operation **does not directly count k-indexed atoms.** It constructs the set of entities which **would be** indexed if the $COUNT_k$ function were to apply to N, and counts that set.

Who has more furniture? = Which set is assigned a higher value on the cardinality scale?
For $x \in$ *furniture* $\wedge$ $y \in$ *furniture*, x is $\text{more}_{\mathbf{CARD},|\,|^{\mathbf{k}}}$ than y iff
$\text{MEASURE}_{\mathbf{CARD},|\,|^{\mathbf{k}}}(x) > \text{MEASURE}_{\mathbf{CARD},|\,|^{\mathbf{k}}}(y)$

Since *furniture* is a mass noun, $\text{MORE}_{|\,|k}$ cannot be used directly since it counts indexed atomic entities.
But $\text{MORE}_{M,U=}$ can be used.
$\text{MORE}_{\mathbf{CARD},|\,|^{\mathbf{k}}}$ is one available instantiation of $\text{MORE}_{M,U}$

Summing up:
Counting assigns a numerical value n to a plurality depending on the number of its atomic parts.
Measuring assign a value $\text{MEASURE}_{DIM,U}(x)= n$ on a scale.
Cardinality scales allow us to use the set of natural numbers as if they were values on a scale. These allow us to assign cardinal values to sums indirectly, without having access to a set of individual atomically indexed parts, and without being able to count them directly.

Note: Pairs of counting values and pairs of measuring values can both be compared but the operations used are different. Thus counting values and measure values can't be conveniently compared.

(23) a. John has more carpets than Bill has curtains.
b. John has more carpeting than Bill has curtaining.
c. #John has more carpets than Bill has carpeting/furniture/curtaining.

(23a) compares two cardinal values at type n i.e. two values on the same scale.
It is true if n is MORE $_{|\,|k}$ than m, where
n = the cardinality of the sum of John's carpets
m = the cardinality of the sum of Bill's curtains

(23b) compares values at type n × U and is interpretable but underspecified since the scale has not been specified (volume? area? cardinality?)

(23c) is the least felicitous, since it compares a value at type n (the cardinality of the sum of John's carpets = n) with a measure value of type n × U (the measure of the sum of Bill's carpeting/curtaining is <n, k>.

8. Counting vs Measuring

If we have cardinal scales, can we reduce all counting to measuring? No.

- N, the set of natural numbers, is an ordered set of entities at type n.
- Cardinality scales are triples $S_{\mathbf{CARD},\,|\,|}{}^{\mathbf{k}} = \langle \mathbf{N}, \geq_{N.} |\,|^{\mathbf{k}} \rangle$

They use the set of natural numbers (and the natural successor relation) to build the scale, with a 'dummy' dimension, CARD and a contextually dependent unit of calibration derived from k.

Counting is an operation which uses the set of natural numbers as primitive objects. It assigns a cardinality to a sum in context k directly by assigning the set of its atomic parts to an equivalence class. The cardinality of x is the cardinality of the sum of its atomic (k-) parts $\{y: y \sqsubseteq_{k\text{-ATOM}} x\}$

Measuring assigns a cardinal value <n,k> to a sum by indirect counting. The cardinal value <n,k> is assigned to x on the basis of the cardinality of a related set $\{y: y \sqsubseteq x\} \cap k$;
The atomic parts of x are not directly accessed.

Cardinal scales are not 'true' measures. The scales are inherently non-continuous, the dimension is arbitrary. However, they share an essential property with all measure operations: they assign a value to a sum without it being specified how that value is a function of its part-of structure.

9. Approximation

If cardinality measures do not require directly accessing atoms, then they are natural methods for giving cardinality values by mechanisms other than counting.

Natural operations which do this are approximation, more precisely estimation. Estimating is giving an overall 'ball-park' cardinal value to a sum without counting. Estimated values are naturally quoted in round numbers, they are values 'in the region of'. For explanations of why round numbers are natural approximate values, see Krifka (2009), Solt (2015).
There are a number of approximation operators and they may involve different approximation operators (24), including modal operators such as *maybe*.

(24) a. about 500 yards away, approximately 5 miles from here, roughly/around about an
b. approximately 500 people, maybe 500 people, roughly 500 people

Russian has a dedicated grammatical operation to express approximation, namely approximative inversion:

(25) a. ona napisala desjat' knig		b. ona napisala knig desjat'	
she wrote ten books		she wrote books ten	
'She wrote ten books.'		'She wrote about ten books.'	

Some approximation clearly involves counting:

(26) a. I counted about 250 people in the room.
b. I made a crochet chain of about 70 stitches.

However, not all approximation derives from the same mechanisms. Approximation in counting contexts like (26) may well derive from perceived inaccuracy in counting rather than estimation. This correlates with the fact that in contrast to clear measure and estimation contexts, precise numbers are more acceptable:

(27) a. ?The station is about three hundred and eight meters walk from here.
b. ?I have about 308 volumes of law reports on my shelf.
c. ?There are about eleven people in my class.

However the following are all perfect:

(28) a. I counted about three hundred and eight privates on the ground…[1]
b. Reasons for admission to asylums: 1864 to 1889. I counted about 28 I'd qualify for…[2]
c. I counted about 11 seconds that passed between the sound and the notification badge appearing in the upper right hand corner of the screen.[3]

The approximation in (28) seems to derive from uncertainty about the accuracy of the counting, and not from the inherent inaccuracy of measuring.
Other approximative values are the result of indirect counting:

(29) Ten rows of fifteen chairs is a hundred and fifty people.

So I am not making the claim that all approximative values need to be expressed via a measure function.

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[1] *Leaves from the Journals of Sir George Smart* George Smart, edited by H. Bertram Cox and C.L.E. Cox, CUP 2014. page 203
[2] https://www.pinterest.com/pin/426645764680043040/ Accessed 26.05.2016
[3] https://discussions.apple.com/thread/7296278?start=0&tstart=0 Accessed 26.05.2016

However, estimation is an operation which explicitly avoids individuating atomic parts.

Furthermore: in a number of languages approximative values are expressed in constructions which have the syntactic properties of measure expressions:

- Russian approximative inversion: Khrizman and Rothstein (2015), Matushansky (2015) show that AI constructions are measure constructions.
- A second construction which shows this is round-number use in Mandarin. (Li and Rothstein 2012)

Mandarin estimation/round number contexts.
In Mandarin counting involves sortal classifiers:

(30) san ge xuesheng
three Cl student
'three students'

[Num Cl *de* N] phrases are interpreted as measures and not counting (Cheng & Sybesma 1998).

(31) a. san bang (de) rou [mass classifiers]
three Cl-pound DE meat
'three pounds of meat'
b. san ge (*de) xuesheng
three Cl (*DE) student
'three students'

de is ruled out in counting contexts (Li 2011, Li and Rothstein 2012).

(32) a. fuwusheng kai le san ping (*de) jiu, yi zhuo yi ping.
waiter open PFV three Cl-bottle DE wine one table one Cl-bottle
'The waiter opened three bottles of wine, one bottle for one table.'
b. ta he le san ping (de) jiu.
he drink PFV three Cl-bottle DE wine
'He drank three bottles of wine.'

However, *de* phrases are possible with sortal classifiers with high round numbers (Tang 2005, Hsieh 2008) and approximative contexts. Examples taken from PKU Corpus via Hsieh 2008:

(33) a. mingtian de huodong xuyao yi bai zhang de fangzuozi.
tomorrow mod activity need one hundred Cl-piece DE square table
'Tomorrow's activity needs one hundred square tables.'
b. nabian bian zhong le qi ba ke, shi lai ke de juzi shu.
there then plant PFV seven eight Cl, ten around Cl DE mandarin tree
'On that side were planted seven or eight, or around ten mandarin trees.'

The *de* shows that these constructions involve measuring. We assume that the measure operation is **estimation** (31) measures the quantity of tables to be equivalent to 100 (k-dependent) *zhang*- units. Estimation does not individuate atomic parts, but gives an informed guess about the overall cardinal value to be assigned to a sum. Exact figures are infelicitous because they can be acquired only by counting and not by measuring.

(34) a. #women xuyao yi- bai- ling- ba zhang de fangzuozi.
we need one-hundred-zero-eight Cl DE square-table
'We need one hundred and eight square tables.'
b. #yinian zhongzhi-le yi- bai- sanshi-qi ke de shumu.
one-year plant-PFV one-hundred-thirty-seven Cl DE tree
'(They) planted one hundred and thirty seven trees a year.'

So there is clear grammatical evidence of the association between approximation/estimation and measurement, and the dissociation between the use of sortal classifiers in counting contexts (where *de* is disallowed) and the use of sortal classifiers in estimation contexts, where *de* is obligatory.

10. Why do we not measure count nouns or compare substance mass nouns in terms of cardinality?

The theory that I have been presenting suggests that the choice between cardinal evaluations and comparison along continuous dimensions for object mass nouns is a matter of contextual preference and not semantics.
So why is this contextual preference restricted to object mass nouns? Bale and Barner 2009 point out that *who has more stone?* seems always to get a volume/weight dimension, while *who has more pieces of furniture*, always has a cardinal evaluation. Why?
This is particularly puzzling because, in Hungarian and Brazilian Portuguese comparisons such as *who has more stone/book* using the mass noun allow both cardinal and non-cardinal evaluations just like *who has more furniture?*

Suggestion: you can get both cardinal evaluations for substance mass nouns and non-cardinal evaluations for count nouns:

Cardinal evaluations for mass nouns in English.
English has for the most part an 'either – or' system: either a mass noun is available or a count noun is available. Count nouns privilege cardinal evaluations because they encode and make salient the (semantically) atomic structure. With *furniture* nouns cardinal and non-cardinal evaluations are available since the predicate is generated by a set of discrete entities, but atomic structure is not encoded.

With substance mass nouns cardinal evaluations are available when discrete portions are made salient, and counting individual portions is contextually relevant.
Assume two field. Field A has a moderate number of middle size and large stones in it. Field B has a very large number of very small stones, so the volume is overall less.

(35) John and Bill are both being punished for some crime or another with community service which involves clearing stone from public terrain. John has four hours of community service and Bill has only two hours.
So John was assigned to the field which had more stone to pick up.
Which field was he assigned to?

Non-cardinal evaluations for count nouns:
These are also available, particularly when the plural is interpreted as a superordinate:

(36) Le-mi yeš yoter taxšit-im
To who there.is more jewel.PL
"Who has more jewellery?"

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